Azonosító jel: | | | | | | | | | | | | | | | |

ÉRETTSÉGI VIZSGA • 2022. május 3.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

2022. május 3. 9:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. **a)** A fair gambling die is rolled 7 times and the numbers shown are added. How many different sequences of 7 rolls are there where the sum of the numbers shown is 9? (The order of rolls is considered important.)

b) A fair gambling die was rolled 8 times. The numbers shown on the first seven rolls were 2, 1, 3, 5, 4, 3, 5. What could the number shown on the 8^{th} roll be if the mean of the 8 numbers was greater than their median?

c) A fair gambling die is rolled twice. What is the probability that the number shown on the second roll is greater than that shown on the first roll?

a)	4 points	
b)	6 points	
c)	4 points	
T.:	14 points	

2. **a)** Statements A, B and C are given. The logical value (truth value) of statements A and B is true, the logical value of statement C is false.
Determine the truth value of the following statements. (In **this** question you are not required to justify your answer.)

(1) $A \wedge C$
(2) $\neg A \vee B$
(3) $B \rightarrow C$
(4) $(A \wedge \neg B) \vee C$

Let x and y be the first and second coordinates of an arbitrary point in the coordinate system and let c be a real number.

b) Is the following statement true?
If $c \leq 12$, *then* $x^2 + 4x + y^2 - 6y + c = 0$ *is the equation of a circle.*
(Explain your answer.)

c) State the converse of the above statement and determine if the converse is true or false. (Explain your answer.)

a)	3 points	
b)	4 points	
c)	3 points	
T.:	10 points	

3. The lengths of the three sides of a triangle (in increasing or decreasing order) form three consecutive terms of a geometric sequence. Two sides are known, one is 12 cm, the other is 27 cm long.

a) How long may the third side be?

The legs of the right triangle *ABC* are *AC* = 30 units and *BC* = 40 units. Draw the altitude, the angle bisector and the median through the right vertex. They intersect the hypotenuse in points *P, Q* and *R*, respectively.

b) Give the ratio *AP* : *PQ* : *QR* : *RB* using whole numbers only. Use exact values!

a)	5 points	
b)	8 points	
T.:	13 points	

4. The efficiency of various means of transportation is often studied. A typical method of comparison is to calculate the fuel costs of transporting 1 passenger to a distance of 1 km. The average fuel consumption of a Boeing 737-700 passenger jet on the 1200 km flight between Budapest and Amsterdam is about 2.4 tons per hour. The average speed of the aircraft is 750 km/h and it carries a maximum of 150 passengers. Jet fuel costs 900 euros/ton.
The fuel consumption of a passenger car is about 6 litres/100 km, and the car carries up to 5 people. Fuel for the car costs 1.2 euros/litre.

a) Assume that both the airplane and the car carry the maximum number of passengers. Based on the fuel cost only, would the plane or the car be less expensive to carry a single passenger to a distance of 1 km?

A certain flight offers sandwiches, soft drinks and coffee for sale on board. A sandwich costs 3.50 euros, a soft drink costs 3 euros, a coffee is 2.50 euros. A menu, that contains one sandwich and a soft drink, is sold for 5.50 euros.
On a particular flight 28 cups of coffee were sold. The number of sandwiches sold in a menu was twice as many as the number of sandwiches sold separately. The number of soft drinks sold in a menu was 10 less than the number of soft drinks sold separately. Exactly one third of the revenue came for selling menus.

b) Calculate the total revenue on this flight.

a)	7 points	
b)	7 points	
T.:	14 points	

II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

5. The three sides of a triangle are a, $a + 1$ and $a + 2$ units long.

a) Prove that for γ, the greatest angle of the triangle, $\cos\gamma = \dfrac{a-3}{2a}$ is true.

b) Give the length of each side of the triangle, given that the greatest angle is 120°.

The three sides of a right triangle are 8 cm, 15 cm and 17 cm long. Select one point (inside or on a side) of this triangle at random.

c) Calculate the probability that this point is at least 3 cm away from each vertex.

a)	6 points	
b)	3 points	
c)	7 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

6. A factory produces pots that hold 5 litres. The shape of the pots is approximately cylindrical with an open top.

a) Find the radius of the base circle of the 5-litre pot, given that its height is 15 cm.

b) The outer surface of the pots is covered in a thin layer of red enamel. What should the radius of the base circle of the 5-litre pot be to minimise the amount of enamel needed to cover the outer surface?

There is a certain probability p that any pot (independent of any other pot) is damaged. A truckload of several thousand pots was delivered to a customer. 20 of the delivered pots were inspected by Quality Control before accepting the shipment.

c) What is the maximum value of p, given that the probability that none of the 20 inspected pots are damaged is at least 0.8?

a)	3 points	
b)	8 points	
c)	5 points	
T.:	16 points	

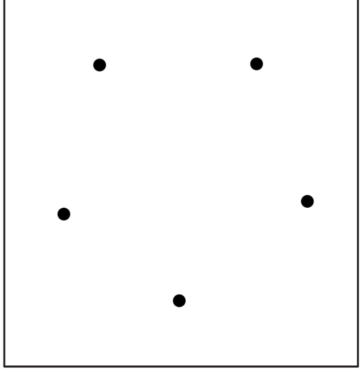
You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

7. **a)** Two positive integers are co-primes (relatively primes), their least common multiple is 35 700. Determine the number of such pairs of co-primes.
(Pairs (a, b) and (b, a) are not considered as different.)

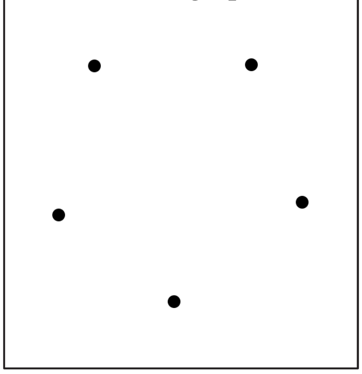
b) Let $H = \{1; 2; 3; 4; 5; 6; 7; 8; 9; 10\}$. How many subsets of H do exist in which the product of the elements is divisible by 9? (In case of single-element subsets the "product" is the value of that single element.)

c) Five points are given on a sheet of paper. A positive integer is written next to each point. Let these points be the vertices of a 5-point graph. Two vertices are connected with an edge if and only if the number written next to one of the points is a multiple of the number written next to the other.
There are five points shown in each of the three diagrams below. In each of the three diagrams write **different** positive integers next to the five points and draw the associated graph according to the rule above such that the first diagram shows a complete graph, the second shows a tree graph and the third shows an empty graph (an empty graph has no edges at all).

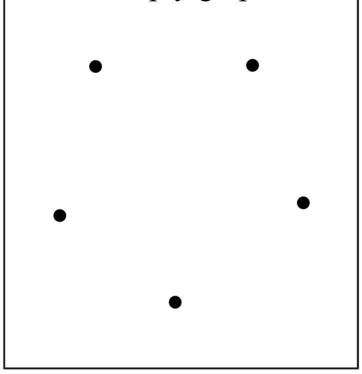
1. Complete graph



2. Tree graph



3. Empty graph



a)	5 points	
b)	5 points	
c)	6 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

8. Cars, taking part in traffic, often have to stop suddenly. Typical deceleration for cars on a dry road is around 7.5 m/s^2.
In this case instantaneous velocity is described as a function of the distance travelled by the formula $v(x)=\sqrt{v_0^2-2\cdot 7{,}5\cdot x}$, where x is the distance travelled, in metres, since the beginning of the braking, while v_0 is the velocity of the car, in m/s, when the braking begins.

a) A car is travelling (on a dry road) at a velocity of 18 m/s when it starts braking. A ball rolls out onto the road in front of the car 20 metres ahead of it. Will the car be able to come to a full stop without hitting the ball?

b) When an accident happens inspectors often measure skid marks (tracks left behind by skidding braked tyres) to estimate the speed of the car just before it started braking. A car (travelling on a dry road) left a 40-metre skid mark between the points where it started braking and where it came to a full stop. What was the speed of the car, in m/s, when it started braking?

The distance a car travels between the points where the driver perceives an obstacle on the road and where it comes to a full stop is called **total stopping distance**. It consists of two components: **the distance travelled during the reaction time of the driver** and the **braking distance**.
The reaction time of the driver is the time interval between perceiving the obstacle and the beginning of braking, during this time the car travels at a steady speed. The distance travelled between the start of braking and coming to a full stop is the braking distance. Deceleration on a wet, icy road decreases to 1.5 m/s^2. In this case the formula that describes the instantaneous speed changes to: $v(x)=\sqrt{v_0^2-2\cdot 1{,}5\cdot x}$.

c) Assume that the reaction time of the driver is 0.8 seconds. Calculate the **total stopping distance** for a car that travels on a dry road at a speed of 15 m/s (54 km/h). What should the speed of the car be for the same total stopping distance when travelling on a wet, icy road?

a)	4 points	
b)	3 points	
c)	9 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

9. **a)** Determine the values of the real parameters a, b and c for the function f: $\mathbf{R} \rightarrow \mathbf{R}$, $f(x) = x^3 + ax^2 + bx + c$ if the following are given:

(1) $f(0) = 1$;
(2) $f(1) = 0$;
(3) $f'(2) = f''(1)$ (the first derivative of f at $x = 2$ assumes the same value as the second derivative of f at $x = 1$).

b) Prove that the graphs of the functions $y = x^3 - 4x^2 + 2x + 3$ and $y = x^3 + 3$ intersect in two points and calculate the area of the region in between these graphs.

a)	10 points	
b)	6 points	
T.:	16 points	

	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	14		**51**	
	2.	10			
	3.	13			
	4.	14			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

	pontszáma **egész számra** kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum

dátum

javító tanár

jegyző